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At the end of  $t$  seconds the co-ordinates of  $C$  will be given by  $x = R \cos \theta - R\omega \sin \theta \cdot t$  and  $y = R \sin \theta + (v + R\omega \cos \theta)t$ .

Eliminating  $R$ ,  $y = \frac{\tan \theta + \omega t}{1 - \omega t \tan \theta} \cdot x + vt$ , or  $y = \tan(\theta + \tan^{-1} \omega t)x + vt$ .

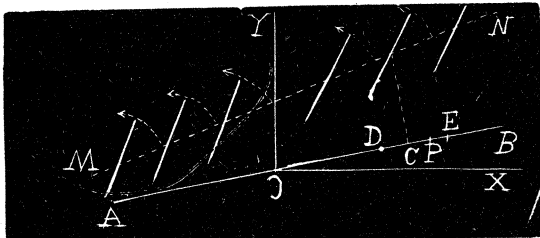
The locus of the centers of gravity of the parts  $t$  seconds after rupture is, therefore, a straight line inclined to  $X$  at an angle  $\theta + \tan^{-1} \omega t$ , and cutting  $Y$  at a distance  $vt$  from  $O$ .

This line coincides with  $Y$  after  $\frac{\cot \theta}{\omega}$  seconds.

When  $t = \infty$ , the locus is perpendicular to  $AB$ .

In the figure the line  $MN$  represents the locus, and the arrows the direction of rotation of the parts. The center of gravity of each part moves uniformly in a straight line forever, while the part rotates uniformly about this center of gravity.

This problem was also solved by F. P. MATZ.



## PROBLEMS.

29. Proposed by J. A. CALDERHEAD, A. B., Superintendent of Schools, Limaville, Ohio.

Show that if a body be projected from the angle  $A$  of a plane triangle  $ABC$  so as to strike the side  $CB$  at a point  $D$ , then, if its course after reflection at  $D$  be parallel to  $AB$ ,  $\tan DAB = \frac{(1+\epsilon)\cot B}{(1-\epsilon)\cot^2 B}$ .

30. Proposed by WILLIAM HOOVER A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

$P$  is the lowest point on the rough circumference of a circle in a vertical plane at which a particle can rest, friction being equal to the pressure; to find the inclination of the radius through  $P$  to the horizon.

## DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

## SOLUTIONS OF PROBLEMS.

23. Proposed by J. M. COLAW, A. M., Principal of High School, Monterey, Virginia.

Find three positive integral numbers such that the product of the first and the sum of the others is a square and the sum of their cubes is a square:

**Solution by J. W. NICHOLSON, A. M., LL. D., President and Professor of Mathematics in the Louisiana State University, and Agricultural and Mechanical College, Baton Rouge, Louisiana.**

Let  $x^2$ ,  $2(n^2-1)x^2$ ,  $2(n^2+1)x^2$ , be the three integers. The first condition gives  $4n^2x^4 = \square$ ; hence we have only to solve

$$\left[ x^2 \right]^3 + \left[ 2(n^2-1)x^2 \right]^3 + \left[ 2(n^2+1)x^2 \right]^3 = \square,$$

$$\text{or } x^6 \left[ 16n^6 + 48n^2 + 1 \right] = \square,$$

$$\therefore 16n^6 = \left( \frac{48}{2} n^2 \right)^2; \text{ whence } n^2 = 36; \text{ and the three integers are } x^2,$$

$70x^2$  and  $74x^2$ , where  $x$  is any integer.

Also solved by H. W. DRAUGHON, and G. B. M. ZERR.

**24. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

*Solve generally:* The sum of the cubes of  $n$  consecutive numbers is a square. Determine the numbers, when  $n=2$ ,  $n=3$ ,  $n=4$ , and  $n=5$ .

**I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.**

Let  $m$ ,  $m+1$ ,  $m+2$ ,  $m+3$ , etc., represent any consecutive numbers the sum of whose cubes is to be taken.

Solving by the differential method, we obtain

$$S = \frac{n}{4} \left\{ n^3 + (4m-2)n^2 + (6m^2-6m+1)n + 4m^3-6m^2+2m \right\}.$$

This reduces to  $S = \frac{1}{4} \{ [n(n+2m-1)]^2 + 2m(m-1)[n(n+2m-1)] \}$ , (A).

If the sum of the consecutive cubes is to be a square, then

$$[n(n+2m-1)]^2 + 2m(m-1)[n(n+2m-1)] = \square = a^2.$$

Adding  $[m(m-1)]^2$  to both members, we have

$$[n(n+2m-1) + m(m-1)]^2 = a^2 + [m(m-1)]^2.$$

This is of the form  $(p^2+q^2)^2 = (2pq)^2 + (p^2-q^2)^2$ .

Equating the respective values, and reducing for  $m$  and  $n$ , we obtain

$2m = 1 + \sqrt{4p^2 - 4q^2 + 1}$ , and  $2n = \sqrt{4p^2 + 4q^2 + 1} - \sqrt{4p^2 - 4q^2 + 1}$ . It will be observed that each of the radical quantities is of the form of an odd square,  $4p+1$ .

There are two conditions that will render the radicals rational, and, at the same time, have  $a$  an integer:—

(1) When  $4q^2 = 4p$ . Then  $m = p = q^2$ , and  $n = 1$ . According to this condition there is but *one cube* that can be taken at one time, and hence there would be no *sum of cubes*. This cube is the cube of a square, and is, therefore, also the square of a cube.

(2) The second condition is when  $p^2 = q^2$ . Then  $m = 1$ ; and substituting this value in (A), we obtain  $S = \left\{ \frac{n(n+1)}{2} \right\}^2$ , which is the square of the sum of the series,  $1+2+3+\dots+n$ . From this, then, we have  $1^3+2^3+3^3+\dots+n^3 = (1+2+3+\dots+n)^2$ , or *the square of the sum of the first  $n$  natural numbers is equal to the sum of their respective cubes.*